CERTAIN UNITIES IN SCIENCE

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THAT the several sciences taken as a whole form one science is a proposition which has often been urged, sometimes apparently as an article of faith and sometimes as a reasoned conclusion. To an individual who holds, with nearly religious fervor, the doctrine that the universe is one and that the truth of science asymptotically approaches the absolute truth about the universe, there can be no doubt of the oneness of all science; there is no room or opportunity, except through error, for that diversity which destroys the oneness of the whole. To an individual of such an outlook it may be almost or quite an article of faith that all science is one. But to him whose universe is not so tidy, in whose thought there is the ever-present possibility that after all we may be building on insecure foundations, the assertion of the unity of science can be made only on a reasoned analysis of its characteristics and on the established fact of the presence in it of such dominant qualities as bind the whole indissolubly into one. To exhibit such elements and to show that they have such qualities is a task of large proportions, for beyond the possible achievements of a single paper. Our purpose is the more modest one of exhibiting certain common elements, certain unities, in science as a whole and of partially analyzing the way in which their presence affects the character of scientific truth.

The unities in science, however far-reaching, can never be absolute. Whatever is common to two domains of knowledge appears in each of them colored by the dominant light of the particular discipline. Perhaps the most obvious unity of all is that of experimentation and observation; its presence in natural science is almost universal. But in mathematics it is partially obscured from view by a universal insistence upon logical connection in exposition, so that the processes of experimentation and observation which were employed in discovery are not in evidence in the finished product. In the case of empirical theorems stated as conjectures (such as have occurred frequently in the history of the theory of numbers) we have the most notable partial exception to what is the general rule. The results conjectured are genuine empirical theorems. Mathematics differs from the natural sciences in refusing to accept these conjectured theorems without a logical demonstration. In thousands of cases it has been observed,

for instance, that an even number is a sum of two primes and no even number is known which does not have this remarkable property. It is conjectured to be true that every even number is a sum of two primes. But, as long as a logical demonstration is wanting, no mathematical memoir or treatise will assert its truth. Such an ideal of carefulness is possible as a practical ideal for the control of actual mathematical exposition; and, since it is possible, we insist upon it absolutely. But, from the greater complexity of its problems and the nature of the truth with which it deals, natural science can not insist upon such perfection of logical form but must rely upon incomplete induction from particular observations to general laws and their subsequent experimental verification either directly or through the intervention of their consequences. The nearly universal unity of experimentation and observation is seen in varying colors in the different sciences.

Probably a more important, if less obvious, unity is that of invention or creation. This is most clearly in evidence in pure mathematics; but an examination of it as it appears there leads to the judgment that it is perhaps dominant throughout all science. There are those who wish to have the universe so tidy that nothing actually novel could happen in it, that happening would be impossible, that every event should be a mere consequence of the events which have preceded it. But there are others who would not object to the surprises and thrills of true novelty, who would not be disconcerted by the conclusion that a law reached inductively by the mind is essentially a creation of the mind, made (to be sure) for the purpose of relating in thought a class of observed phenomena, but none the less a veritable creation or invention. The whole matter turns on this question: Are the laws obtained by induction found in nature and dictated entirely by nature, or does the mind in some manner impose its own bias upon them? It appears that we are forced to the latter conclusion, particularly when we see the dominance of a general theory like the atomic theory or the rapid inroads of such a one as the theory of relativity and reflect how these were conceived in the mind before there existed any empirical evidence for them. In such cases as these the mind has either imposed its own bias upon the laws of nature or it has had an uncanny foresight of them before they appeared in experimental science. The former seems to be the more natural and justifiable hypothesis.

There is much to be said in favor of the thesis that natural science should be considered a construct of mind rather than a paraphrase of nature wrought out by the mind. The processes of invention are most in evidence in the formulation of hypotheses, and most clearly when these are based on only a few observations. There is no experimental proof, and perhaps in the nature of things there can be none, for the

hypothesis of the conservation of energy on which all modern physical science is based. This seems to be a law imposed by the mind for its convenience but without direct experimental support; at best it is contradicted by no experimental fact.

But invention seems also to be present in the process of experimentation. The experimenter is not a merely passive recipient. He is active in directing the course of events. He invents phenomena which would be non-existent without his guiding influence. He gives attention to what he wills and ignores other things. He will not see all that happens, nor will he record all that he sees. He selects before he places on record for the examination of others.

The principle of direct causality is almost universally held to underlie all natural science; the principle of inverse causality is also generally asserted as true, but with less confidence in the assertion. If the principle of causality exists at all in mathematics it must be in some greatly attenuated form. It can hardly be said that the triangularity of a Euclidean triangle is a cause having as an effect the proposition that the sum of the angles of the triangle is equal to a straight angle—unless one thinks of the cause-and-effect relation as having here a quality peculiar to mathematics. But in the natural sciences the principle seems to rule supreme. In some of them it is employed mostly in the direct sense, as in physics where one generally utilizes it for proceeding from the cause to the effect; in others the principle of inverse causality is more often in evidence, as in geology where we infer the past state of the earth from its present state. If the principle of causality affords a unity in science it can do so only on the assumption of at least the three well-distinguished forms which we have just mentioned. Moreover, however one approaches it, it involves him in speculative difficulties from which it is hard to extricate himself. It is a severe, if not an impossible, task to adjust his conception of the principle and his practice in its use so as to avoid just criticism and his own dissatisfaction with it.

The difficulties which we have seen here in these cases are probably to be found, singly or in combination, in the case of most of the more obvious unities in science. The element of unity may fail to be as well marked as we like throughout the whole range of the sciences, as in the case of the unity of experimentation and observation; or it may be of such character that people can not be brought to general agreement about it as in the unity of method involved in the hypothesis of invention or creation; or it may lack somewhat in oneness, as in the case of the three forms in which the principle of causality appears. The main object of this paper is to discuss certain actual or possible unities not having these defective qualities.

The complexity of nature is great beyond our ability to understand

or perceive. The material universe is too rich in form and the fullness of phenomena for us to reach the whole extended complex in a single grasp of the mind. The extreme variety of kinds of objects, the multitudes of individuals of a kind, their almost innumerable relations in time and space, the ramifying causal connections among them and their mutual dependencies, their diverse relations to our own life and thought, and the hidden things in them which our organs of sense are unable to perceive even when supported by the powerful instruments of science—all these tend to produce a complexity in the presence of which we are helpless so far as logical organization of all impressions is concerned.

Even in the realm of those objects of thought which are constructed by the mind itself there is too much diversity for us to contemplate the whole at once if we are to do anything other than make glib general statements unsupported by anything more than a certain appeal to the imagination. It is evident that the mind is able to contemplate successively the elements of a range of objects of thought far too vast to be embraced in a single encircling mental act. This is true not only in general but also in the case of such extended ranges as pertain to a single domain, as for instance that of mathematics or that of philosophy.

Two quite distinct worlds about which we should have exact information may be conceived separately; the world of matter and the world of logical thought. Let us examine the two things presented to our consideration by the physical phenomena of matter on the one hand and on the other hand that special domain of logical thought which is embodied in mathematics. There is a very wide range of mathematical knowledge apparently unconnected with the properties of matter. There are physical properties of matter, so complicated that mathematical methods are still powerless in their presence. Each of these domains is vast in its extent. There is a relatively narrow strip on which the two overlap, the properties of matter yielding themselves to mathematical formulation and the mathematical truth seeming to have its concrete embodiment in the properties and phenomena of matter. The existence of this common region of the two things apparently so widely separated has arrested our attention and has directed it so forcibly to the striking parallelism that we have sometimes felt that we have in it a fair measure of evidence for believing the whole universe to be rational. So far this conclusion has too much the appearance of a pious wish and two little the character of a demonstrated result to justify our confidence in it. The relative narrowness of the common region of the two is rather disconcerting if we examine it closely. Even if all physical relations should be reduced to mathematical formulation we would still have far to go to

reduce all phenomena to rational order and to find logical connections among all their parts.

This diverse character of the most widely separated elements of physical and of mathematical science is one evidence of the necessity for breaking up into parts the total body of material concerning which we seek to attain exact and permanent knowledge so as to bring it within the range of such methods as we are able to conceive and employ in one connected investigation or analysis. But if we break this material up into parts it is only by ignoring certain connections of importance, only by making abstraction of elements which may be omitted for the intended partial view but are essential to a complete understanding of the whole.

The general situations actually presented by nature or by thought are too complex for us if we are to gain permanence or invariance in the conclusions which we reach. We have to create ideal situations where we are more at home and over which a restricted range of method will carry us with safety and with conclusions of sufficient penetration to have abiding value. We have to adapt our procedure to the strength of tool afforded by our minds when brought to their state of highest effectiveness. With a more penetrating insight less abstraction would be necessary; but only omniscience would enable us to conceive and handle at once the total flux of nature and thought. We have to work subject to the restrictions of our essential limitations.

This process of abstraction has been carried further in mathematics than in any other science, having attained a place of importance there long before its primary character was recognized in other disciplines. Every organism possessed of locomotion has to deal with the problem of space relations, and particularly an architectural animal like a beaver or a man. Long experience in construction and measurement will give rise to a certain body of empirical knowledge and rule-of-thumb methods for making standard constructions. To such a state of advancement the knowledge of space had already attained among the ancient nations of the Orient and particularly among the ancient Egyptians. But their progress was intercepted by their inability to make needful abstraction of the essentially irrelevant and to concentrate on those properties which afford the essential elements of geometrical form as such. They could only imperfectly conceive a triangle as anything more ideal than a piece of land of a certain outline or a flat stone of a certain shape.

As long as the problems are conceived as those of the space relations of material objects there is present to thought a large disturbing element which successfully turns the attention away from what is essential. In order to construct a theory of the space relations of objects it seems to be practically necessary to do a more ideal thing

first. Before one can make serious progress in the way of definite conquest, one must abstract from the general complexity of the situation and attain to a new one relatively much simpler. In fact, not only in the study of properties of geometrical space but also in many domains of science it is necessary to create a new situation having certain analogies with the actual one of nature but being so much simpler that we are able to grasp the interrelations of all its parts.

This idealizing of the problem of space relations was first effectively achieved in the geometry of the ancient Greeks. They were able to get away quite completely from the material triangle and to conceive the ideal triangle defined by certain essential ideal properties. Likewise they were able to make abstraction of what was not necessary to the purposes of a pure geometry in the various lines and circles and other figures which they wished to consider. This new attitude toward the subject matter of the theory of geometrical space allowed an altogether unforeseen extension of knowledge; geometry came into being in one of those forms which stand as part of the modern theory. By abstraction of unessential elements the mind came to behold a much simplified object of thought and analysis a knowledge of whose properties gave the needful insight into the space relations involved in normal everyday experience.

For a long time this body of geometrical truth stood apart from all other knowledge, separated by qualities of generality and ideal conception from all other doctrines whether of mathematics or of some other discipline; but, after a time, algebra began to assume a like position of separate completeness and it existed so until algebra and geometry were brought together by the invention of analytical geometry.

The abstraction of the unessential in the study of space relations, difficult as it was and effected only in relatively recent times, seems to have been the easiest large abstraction for the human race to achieve. This was probably due to our intimate racial acquaintance with the space of experience during the whole period of our evolutionary history and to some peculiar adaptation of ourselves to the understanding of spatial relations. That our long drawn out experience with it is not in itself sufficient to enable us to fix attention upon the essential elements and to understand their relations is shown by the fact that we have been quite as long acquainted with the weather as with space relations and that we have not yet been able to reduce to the form of an exact science our knowledge of its daily changes—unless indeed we have been hindered in such progress by essential changes in the character of the weather during geological ages while the relations of space have been an invariant element throughout our experience.

The fact that mathematics first succeeded in making these large

abstractions from the complexity of the environment in building up its body of doctrine and that is today relative the furthest advanced of the sciences raises the question as to whether there is a general correlation between the state of advancement of a science and its success in forming appropriate abstractions. The just conclusion seems to be that no science is far advanced until it has first succeeded in isolating by abstraction a large body of material, conceived ideally apart from the matrix of its environment and possessed of such essential properties as make it possible to pass from the conclusions of this ideal science into the actual complexity of phenomena with a better understanding of important phases of the latter than is otherwise possible.

In meteorology, where successful abstraction is exceedingly difficult, we find a relatively small body of securely achieved truth. The same is true in our study of industrial organization and of the complex phenomena of the social relation. But if we turn to the work of the astronomer in celestial mechanics, where nature herself almost made the abstractions for him, we find a science relatively far advanced and one which achieved its position of preeminence early in the modern era.

Under the inspiration afforded by the laws of Kepler Newton meditated on the question as to the ultimate law of nature upon which the properties of the planetary orbits depend; and he was led to conceive, and establish by geometrical reasoning, the principle of universal gravitation and the law that the force of attraction between two material particles is directly proportional to the product of their masses and inversely proportional to the square of the distance between them. These discoveries of Newton condensed an almost immeasurable volume of thought into a compact and simple formula, bringing the theory and observation of past ages to a focus from which new lines might diverge in many directions. Laplace undertook to draw out the consequences of the laws of Newton, his purpose being to "offer a complete solution of the great mechanical problem presented by the solar system, and to bring theory to coincide so closely with observation that empirical equations should no longer find a place in astronomical tables." His success in both respects brought him very close to his lofty ideal.

The "Mécanique Céleste" of Laplace holds a unique place in the history of science. It was the first instance in which an extensive unified and logical theory had been developed for a large class of observed phenomena. A very few fundamental principles lay at the basis of the entire work, owing to abstraction of unessential details; and it was developed from these principles in its entirety solely by mathematical processes, this logical procedure being rendered possible

by the essential simplicity of the ideal situation. In such a body of truth there is something esthetically satisfying in a high degree. It could not fail to have a profound effect on the development of thought.

The cosmical and subsequently many terrestrial phenomena having been explained, it was natural that Newton, and still more Laplace and his school, should attempt the explanation of molecular phenomena by similar methods, importing into molar and molecular physics the astronomical view which had arisen in mechanics. This celestial mechanics became the model for the exact sciences. Men sought to give to other theories an equally beautiful and logically consistent form. The start from a few principles, easily enunciated and readily comprehended; the forward march of the theory into new fields, comprising in the range of its explanation an ever-increasing portion of observed phenomena; and its ultimate comprehensiveness in this respect—these things gave it a hold on the imagination. It thus became a profound factor in the development of the whole of physical science in its mathematical form. The cold touch of exact thinking and the calculating mind, both products of the method of abstraction in the development of scientific truth, have proved to be the spell by which knowledge has been found, new sciences have been created and a novel trend has been given to the development of thought.

Perhaps I may digress widely enough to indicate how the precarious character of scientific advancement is indicated by some of the matters now in consideration. Suppose that our earth, instead of being one of a few planets moving around a single sun, had been one of several satellites of a large planet moving about the center of gravity of a double star; then it is clear that the facts of astronomical observation which would first have pressed themselves upon our attention would have been much more complex than those of our actual system. When we consider the long period of time which it required the race to unravel the intricacies of the much simpler system of which the earth is a part and the uncertain and haphazard way by which it took the necessary steps, we see that there is room for grave doubt as to whether we would ever have conceived a suitable explanation and whether (having missed such a guide as this) we would be in a position successfully to attack or even to conceive the problems of natural science. Our progress has been made under the inspiration of the ideal afforded by astronomy on account of a success due to abstractions to which the nature of the observations pointed the way. Could we have ever conceived such an ideal if we had been confronted with a much more complicated astronomical system? And could we have already built up, or would we have ever been able to build up, a body

of science comparable to that which we possess today? Certainly the answer can not be a confident affirmative.

Natural science and mathematics are not the only domains of thought in which the principle of abstraction is prominent; it appears also in speculative philosophy. But its relation to the latter is quite different from that to the former domain. In science we are quite willing to admit abstraction frankly as a universal characteristic which is necessary on account of the limited character of the intellect. But the philosopher desires to get away from it as far as possible. He wishes to embrace in his system an ever-widening range of material; and he would be most pleased if it might be ultimately comprehensive. He knows that he has not attained to such an ideal and probably never expects to; but he still feels a certain sense of uneasiness when he finds a body of truth quite unrelated to the system by which he has brought things to order in his own mind.

But in art and literature the matter is quite different. Here the intention is to contemplate at once the whole stream of life and existence, at least so far as to have no purposed exclusions. Here one deals with the actual complexity of events and even with the character and emotions of individuals. One does not state theorems; one does not announce universal laws; one does not reach rules-of-thumb for doing mechanical things; one does not even find general principles upon which there is universal and permanent agreement nor those by which one may have precise guidance for conduct in any given situation; but one does reach permanent conquest in the creation of things of enduring beauty; one produces lasting values for the emotional life even if one does not increase the body of exact knowledge.

From the foregoing considerations it appears that the method of abstraction, as an active means for clearing off the ground and an active support to the consequent investigations, is common to all science and is characteristic of science. It is therefore one of the important characteristic unities in scientific method. We find it necessary to isolate one subject from another by abstraction in order to avoid being smatterers. We reduce our serious problems to ideal abstractions because no deep-lying problem can be solved without reducing it to abstractions. If we do away with them we do away with mathematics and logic and natural science. They have thoroughly justified themselves through the marvelous conquests of modern science which they have so effectively supported.

But these abstractions are not without their dangers. It has been said that the supreme fallacy of the academic mind is carefully to make abstractions and then straightway forget that they are abstractions. "The expert in the conceptions belonging to one field of knowledge legitimately solves the problems of that field in their terms. But some-

times he forgets that these are very special and limited notions of truth, applicable only to that one field. He ignores that his science is only one abstracted aspect of concrete life, separated from other aspects of life only for the sake of specialization of labor. Ignoring this, he attempts to solve the problems of other fields with his own field's special concepts. Thus, a biologist sometimes endeavors to reduce all psychology to biological concepts; or an economist to reduce all moral values to the special values of the economic world."

Perhaps, for the sake of unity in point of view, one may be allowed to treat, as resulting from a certain form of the method of abstraction, a quality of the mathematical formulation of the laws of nature which first appears explicitly in the general Einstein theory of relativity. This is not the place to give any of the technical developments [1] belonging to the latter theory. But it is a matter of general scientific interest to indicate the character of a new ideal for the form of scientific laws which was first insisted upon in the investigations of Einstein, particularly as it can be successfully described without any of the heavy mathematical machinery which is essential to a detailed development of the theory. This ideal emerges in connection with the analysis of the now celebrated principle of equivalence, which we may enunciate as follows: A gravitational field of force is exactly equivalent to a field of force introduced by a transformation of the coordinates of reference so that we can not by any possible experiment distinguish between them.

The notion of transformation of coordinates of reference, which appears here, is quite essential to an understanding of the quality of mathematical formulation of laws which we wish to explain. Perhaps we may best approach the matter by conceiving a geometrical curve fixed in the space interior to a given room of four walls meeting at right angles. If we take the floor and two adjacent walls to be a system of reference by means of which to locate the positions of a point in the room, then we can uniquely define the positions of a point on our curve by giving its distance from the floor and from each of the two walls selected. If the point moves along the given curve then the numbers expressing these distances will be related according to a law determined by the shape and position of the curve; these three variable numbers will satisfy certain equations of condition. If we used the ceiling and the other walls as a system of reference we should in general obtain different equations of condition for the same curve. Those would be further modified if we chose for reference system some other set of three planes mutually perpendicular to each other, and especially so if these planes should be oriented in some new directions.

[1] The reader interested in these developments will find them in the second edition of my "Theory of Relativity," Wiley & Sons, 1920.

It is clear that these changes in the system of reference have in no wise affected the properties of the curve itself, though they have constantly modified the mathemaitcal expressions by means of which we may most compactly and most completely describe the curve and its position. Let us for a moment forget these systems of reference and study the curve itself by passing along it from point to point. Two characteristics will force themselves upon our attention; the amount of bending of the curve as we pass along it, its curvature; the amount of twisting of the curve, its torsion. These are intrinsic properties of the curve itself capable of representation at each point by definite numerical values. These numerical values can be expressed in terms of the three distances pertaining to any given one of the systems of reference mentioned above; it turns out that definite rather simple formulae exist for expressing the curvature and torsion in terms of the named measurements. Since these describe intrinsic properties of the curve their values must be unaltered by the transformations of variables due to the changes of the system of reference; that is, they must be invariants of the transformations.

It is seen therefore that the analytic expressions for the curvature and torsion are unchanged in form and in value as we pass from one system of reference to another. It can be shown that they completely determine the intrinsic properties of the curve. Then we have in them a complete mathematical description of the intrinsic properties of the curve in a form from which we have abstracted those pecularities which belong to the special system of reference by means of which we described the curve and its position in the first place. This sort of abstraction is of frequent and important use in mathematical investigations. It affords one of our methods of excluding from consideration those things which are irrelevant to the central purpose of the investigation and of fixing attention upon those things alone which are unaltered by, or are invariant under, the transformations permissible among the elements in consideration. A similar but extended use of invariants is a central feature of the Einstein theory of relativity.

Two rather considerable extensions of the method are necessary in order to realize the situation in the Einstein theory. The first has to do with a generalization of the system of reference. In what we said above we contemplated the location of a point always through the measurement of its distances from three planes. Now we wish to replace the three planes by three warped surfaces, perhaps twisted and corrugated and bent into a great variety of shapes and restricted only enough to allow us to utilize them successfully for the location of points in space. By means of these we are to describe the space-configurations with which we have to deal.

The other step which we wish to take is that in connection with the

introduction of time into our system. We can not well develop the mechanics of three dimensions by means of what is simply static in three dimensions; and the introduction of motion and the analysis of velocities and accelerations require the use of the time variable. Moreover, we are not to think of time and space as independent but are to consider the two together as furnishing the four-fold extension of a time-space continuum. This gives us, of course, a space of four dimensions; and in this space of four dimensions the movements of the natural world are represented by static figures. In this space of four dimensions we are to choose as a system of reference four warped three-dimensional spaces by means of which the location of points in this four-dimensional space shall be defined.

With these conceptions in mind we shall undertake to make clear the nature of the central ideal upon which Einstein insists. He wishes to have the laws of nature expressed in such form with respect to this four-dimensional continuum that there shall be no change in the form of these laws when we pass from one of these systems of reference to another; the statement is to be an invariant one when all quantities involved are changed in accordance with a transformation which carries us from any one of these systems of reference to another; let us say for convenience that the laws are to be stated in covariant form. When we have put them into such form we have abstracted from the statement whatever pertains to the particular system of reference employed.

It is a grave question whether the laws of nature are capable of formulation under such radical restrictions; and an affirmative answer can be maintained only after a searching examination. The best way for a just trial of it is to employ one of the best established and most satisfactory laws; none could be more suitable than the Newtonian law of gravitation. Hence one of the first efforts of Einstein to test out the theory was made in the attempt to apply it to celestial mechanics. It turned out that the Newtonian law of attraction does not accord with the ideal of covariance of the laws of nature; it is not capable of expression in precise covariant form. Is the principle then to be surrendered? Not without further evidence; it may be, after all, that the law of Newton is not exact.

The next task of the investigator, then, is to inquire whether there is some slight modification of the Newtonian law which will bring it into covariant form without making it false to experimental fact. It was not difficult to show that the Newtonian law was a very close approximation to a law which is indeed covariant; and the latter was then taken to be the law which should replace the Newtonian law.

Questions which force themselves upon our attention then are the following: Does the new form of the law have any definite advantages

over the old? Can it be subjected to an experimental test to determine which of the two approximate laws is the correct one? Now it happens that the Newtonian law has long been known not to agree exactly with observation in the matter of the motion of the planets. In the case of Mercury the discrepancy is altogether too great to be attributed to experimental error. If the law of Einstein is applied to the problem it accounts for all these motions within the limits of experimental error. Here it scores its first victory over the Newtonian law.

A second crucial test of the theory is offered by its prediction of the deflection of a ray of light which passes through a strong gravitational field. This prediction was tested by observations made independently at two stations during the eclipse of the sun of May 29, 1919. The problem was to determine the amount of bending in a ray of light passing near the sun and hence through its strong gravitational field. The values for the deflection obtained at the two stations are 1.61 and 1.98 seconds of angular measure, resulting in fairly good agreement with the predicted value of 1.74 seconds of angular measure.

The ideal of the covariance of the laws of nature as a practical ideal thus passes successfully its first test, and indeed in a dramatic manner. That Maxwell's electromagnetic equations may be reduced to a covariant form and hence that all electromagnetic phenomena described by them are in agreement with the principle of relativity may be readily shown; and thus the ideal of covariance meets a second fundamental test. There is no known case in which it must certainly be surrendered, though there is an important one which remains still in doubt (see p. 105 of my "Relativity" already referred to).

If all the laws of nature can indeed be expressed in covariant form we have through this fact brought to light a certain profound unity in the laws of natural phenomena, one which will surely be satisfying in an esthetic way to every one who contemplates it with understanding.

We have insisted upon the importance of abstraction as a means of bringing the complexity of the phenomena contemplated within the power of the mind for purposes of systematic analysis. There is also another quite as important reason for making abstraction of certain elements involved in the general complex of the environment; and that is the necessity or desire to find a range of phenomena and objects of thought about which there can be at least a fairly good agreement. The propositions concerning which there is general agreement among competent persons are said to have an objective validity; others are called subjective. Upon being pressed for a definition of "objective" as employed in the phrase "objective character of science" the scientist sometimes asserts that the objective is that which pertains to the world which is external to ourselves or to the world of objects whose essential character is not affected by the subject who contemplates it. But if he

is further pressed for a criterion to determine whether a given thing is objective he has to return to the conception of the objective as that about which there is general agreement among competent persons.

It may be objected that this definition describes merely that which is invariant and that we ought to refer to the invariant character of a scientific thought rather than to its objective character. But it seems to me that "objective" as applied to things of science has no scientifically definable sense except that which rests upon the idea of invariance, at least if one admits (as I think he must in matters of science) that a definition should be so stated that it is theoretically possible to determine whether or not a given thing meets that definition. Moreover, "objective," as the word opposed to "subjective," seems to be well suited to convey the connotation desired. At any rate, it is our purpose to proceed from this as a tentative definition to a more penetrating analysis of the ideal of the objectivity of science, an ideal which can be attained in any particular situation only by excluding from consideration a large portion of the attendant circumstances.

Regardless of the way in which we frame the definition it is agreed that an essential quality of scientific truth is its objectivity. It must depend solely upon the object studied and not upon the subject who investigates. It must be impersonal, having validity independently of the temperament or the peculiar disposition of the individual who reports it. What do we mean by such a demand as this? What can we mean? It is clear that the investigator can not be a mere passive recipient of impressions, a tablet on which nature registers her characteristics. He must be active in several ways; he chooses the things to which he shall direct his attention, his reason or intelligence is an essential element of the registering apparatus, and he is restricted by the limitations of his sensory equipment. He can obtain and convey only that information which his nature fits him to acquire and report upon. The demand for objectivity can not be a requirement that he shall do otherwise. But it is a call for the exclusion of the subjective element, the element which is peculiar to his individuality; and this exclusion is to be brought about by such comparison of his report with that of others as shall make it possible to determine those elements which are independent of his individuality.

But it is clear that such a procedure affords us no means of excluding what is peculiar to the human race as such. This would require the existence of many cognate races of widely different social characteristics by the comparison of whose scientific conclusions we could eliminate that which is peculiar to each, retaining as a residue only that which has objective validity relative to the group of races as a whole. Such a procedure, if the means were at hand for realizing

it, would carry us one step further toward the far-off goal of absolute truth. But we shall have to be content to work without such means of removing from our body of knowledge the elements which are peculiar to our racial individuality. This affords no occasion for dissatisfaction, since the only use we have for our science is that which can be realized by human beings. But it is sufficient to assure us that we have no means of reaching absolute objectivity in our science, if such a thing is indeed conceivable.

The conclusions or observations which we shall admit as having the required objective validity are those which are invariant in the sense that they are reached by all normal human beings who investigate properly the pertinent matters. We can not get along without the qualification of normality of the individuals admitted to the group for which we seek the invariance in consideration nor can we omit the requirement of the proper investigation of pertinent matters. It is hard to see how we can altogether remove subjective considerations from the process of determining when these conditions are adequately met. We can not avoid the conclusion that the highest objectivity realizable in practice falls far short of the quality of being absolute. The conclusions and observations which have the requisite objective character are those only which are invariant for a properly determined group of individuals; and the determination of the group to be admitted can be effected only by the members of the group, since there is no external intelligence that sets them apart.

It is convenient to distinguish two types of objectivity, as here defined, differentiated as to range of time through which exists the group of individuals by means of which each is realized. We may call the one contemporary objectivity and the other historical objectivity, the former being asosciated with a group living contemporaneously and the other with a group scattered through a long period of time, say the whole historical period. All the general truth which is universally approved in a given age among people properly qualified to form a judgment upon it possesses this contemporary objectivity. That which meets with acceptance from age to age with unchanging uniformity has the higher order of historical objectivity. From the truth possessing contemporary objectivity that and that alone survives to attain to historical objectivity which impresses itself alike upon the peoples of succeeding ages.

The subjective character of matters of taste is notorious. Even the milder objectivity of the contemporary sort is seldom attained, and always only imperfectly. Universal agreement on such a judgment of values in a given age carries with it no assurance that succeeding ages will concur in the conclusion. Yet there are some judgments concerning matters of art which go far towards exhibiting the qualities of

historical objectivity. There is an abiding unanimity, for instance, in ascribing a high excellence to the finer elements of Greek sculpture. The more magnificent creations of this art impress with their marvelous beauty the people of one age after another; and these all appear to obtain from them a joy of the same general character. It is true that individuals represent this to themselves variously and that they differ greatly when they seek to give an account of the way in which they are affected. But certain elements of the judgment of value seem to be invariant from age to age and from individual to individual. So far as this is true we have a manifestation of objectivity even in these matters of judgments of taste.

If we find thus a measure of objectivity in these things which are usually esteemed to be highly subjective in character, we also find certain elements of subjectivity even in the matters of science and marked elements in some bodies of truth considered objective by those who develop them. Merz, in his "History of European Thought in the Nineteenth Century," a work to which the present author is greatly indebted in several ways, says: "Most of the great historians whom our age has produced will, centuries hence, probably be more interesting as exhibiting special methods of research, special views on political, social, and literary progress, than as faithful and reliable chroniclers of events; and the objectivity on which some of them pride themselves will be looked upon not as freedom from but as unconsciousness on their part of the preconceived notions which have governed them." Thus the objectivity to which these historians have attained appears to be only contemporary in character.

In forming a judgment of the significance of modern science it is important to ascertain the character and measure of objectivity to which it has attained as a whole and to make a classification of it into parts according to the extent of its success in becoming objective. A large portion of what is now current has gained its position so recently and has so forcibly ejected the earlier explanations to make place for itself as to raise a reasonable question of doubt concerning the validity of the whole structure. When theories have changed so constantly, so long and so profoundly, we can not well believe that we have suddenly come to a state of stability. The changes are likely to continue. If they are retarded for a time they will probably break forth later with increased violence. It is not long since we witnessed a period of explosions in the theory of matter and motion; and indeed we do not yet appear to have come to the end of it.

In the midst of this rapid change what permanent truths are to be perceived? Which can maintain themselves through the present generation and achieve historical objectivity through the support of future thinkers? It seems clear that it can not be the theoretical explanations,

except in relatively few instances if indeed in any, at least if the explanation is conceived to carry with it the means of affording a penetrating insight into the phenomena explained. If the theoretical explanations do not abide, what is there left? Simply and solely the account of relations among phenomena, however these are expressed, whether by means of the mere record of observations or through the more powerful tool of scientific theory conceived merely as a mnemonic device and a support to the weakness of the intellect in its deductions. The statement of relations has in many cases attained historical objectivity in natural science; but theoretical explanations have usually suffered change from age to age and the process seems likely to continue.

Mathematical truth, so far as it is expressed in definite theorems, has achieved almost complete historical objectivity. A result once attained abides through the ages. Errors are made with relative infrequency and these are usually corrected with such definiteness as to secure general and abiding agreement. The permanence of result has for a long time been considered one of the essential glories of the discipline. But there is lack of such complete objectivity concerning the character of the truth attained. Our conception of the position of Euclidean geometry in thought and philosophy, for instance, is far different from that of the ancients owing to the existence of the so-called non-Euclidean geometries of relatively recent times.

If we analyze the remoter origins and earlier bodies of thought by aid of the criteria which we are using, we shall find that we can not deny to the proto-science of savages a certain contemporary objectivity even though its explanations are framed in terms which we perceive to be anthropomorphic or mythological. That the "sun is the flaming chariot of the sun-god, driven day by day across the heavens" is an immediate fact of observation expressed in anthropomorphic language; and probably no more was read into this statement of fact than we are accustomed to transport from our theories into our account of what happens during an experiment or formerly into the similar statement that two bodies attract each other. The primitive explanations maintained their place for a long time; and much useful knowledge was acquired through their assistance and much skill was gained in logical analysis before it was possible to prove them insufficient. "A false theory which can be compared with facts may be more useful at a given stage of development than a true one beyond the comprehension of the time, and incapable of examination by observation or experiment by any means then known. The Newtonian theory of attraction might be useless to a savage, to whose mind the animate view of nature brought conviction and helpful ideas, which he could test by experience." We can deny to the savage neither the use-

fulness nor the contemporary objectivity of the proto-scientific explanations which he offered. They were objective to him in every defined sense in which our science is objective to us. If one points out the anthropomorphic element in them, our criticism will at once be hushed by the anthropomorphism of many of our current conceptions, as for instance that of force. If one objects to the mythical element in their thought let him first take up arms against the colossal myth of the ether in the science of the past century and cut off our thought from this fiction of the scientific imagination. As long as we find it necessary to transport into our theories such elaborate creations as that of the ether (brought in without a shred of direct evidence for their existence) we have little room to complain of the thinkers who formed the proto-science of the savage. Measured by the time through which their explanations maintained their contemporary objectivity, the period during which current scientific explanations have held their place is strikingly short.

The objectivity of truth is never absolute, but always relative to a group of thinkers or the age or ages to which the group belongs. We have no means of removing from our knowledge the marks of our racial characteristics or reaching further into our understanding of nature than to those elements which our sensory equipment enables us to perceive. We have to determine with the best standards we may the group of people in relation to whom we shall insist upon the invariant character of truth as recognized. Since at any time in our history future experience is yet to be evolved we can strictly speaking, have only a sequence of contemporary objectivities, as it were, and never a complete historical objectivity. We can have no logically certain means by which to choose securely those elements of thought in any age which make the nearest approach to complete historical objectivity. A subjective element, that is, an element which varies essentially from individual to individual, is necessarily present in every attempt to reach a means of determining what truth has the dignity of an objective character.

The sciences in coming from under the tutelage of philosophy have not completely shaken off the incubus of its unsupported speculations and prejudices. Phenomena are observed through the goggles of philosophical preconceptions, not only in psychology and biology, but also in chemistry and physics, and even in mathematics; and the conclusions or appreciations are affected in various ways and to different extents. In our generation, in the case of physics (the most advanced of the natural sciences) there is going on a veritable revolution in regard to the philosophical preconceptions on which it is based. As evidence we cite the current discussions of space and time and gravitation.

Again the objectivity of a natural science is relative to the character and measure of abstraction through which it was built up and the syntheses by which the separated elements were afterwards brought together and combined into a unity. This process of synthesis can never be carried to completion without the certain loss of objectivity in the resulting knowledge; and as long as it is not carried to completion we have no means by which to be assured that a matter first treated as essentially irrelevant shall not later come into the focus of attention. In fact, this very thing has recently happened in physics. In studying the properties of light physicists were for a long time content to leave out of account the gravitational field as having no appreciable (or even conceivable) influence; but the Einstein theory has forced them to a fundamental revision of this supposition and has led them to conceive of the ray of light as warped out of a straight path by the action of a powerful gravitational field.

The failure of science to obtain completely its universal ideal of objectivity does not diminish our interest in it. Indeed it is rendered more attractive to those of us who are pleased with a dynamic rather than a static world. Truth is never to be set off in tubes hermetically sealed. It is living and hence possesses the universal quality of life of doing the unexpected thing. Its growth is not hemmed in. We may look forward to its continued progress and novelty as long as we who develop it are finite intelligences.